Test Review Algebraic Modelling SOLUTIONS

Note Title 01/11/2008

Note 02/03/2009

1a) $\begin{matrix} & S & M & L & XL \\ \text{tshirts} & 20 & 60 & 40 & 20 \\ \text{s.pants} & 15 & 50 & 20 & 5 \end{matrix} = A$

b) $\begin{matrix} \text{t-shirts} & \text{spants} \\ \$ & 15 & 20 \end{matrix} = B$

c) $B \times A = \begin{pmatrix} 15 & 20 \end{pmatrix} \begin{pmatrix} 20 & 60 & 40 & 20 \\ 15 & 50 & 20 & 5 \end{pmatrix}$

$= \begin{pmatrix} 15(20)+20(15) & 15(60)+20(50) & 15(40)+20(20) & 15(20)+20(5) \end{pmatrix}$

$\begin{matrix} & S & M & L & XL \\ = \$ & 600 & 1900 & 1000 & 400 \end{matrix}$

Total: $3900

2a) $\begin{matrix} & M & CC & WB & S & KC \\ \text{Sept} & 80 & 50 & 30 & 25 & 125 \end{matrix} = A$

b) $\begin{matrix} & M & CC & WB & S & KC \\ \text{Oct} & 35 & 20 & 14 & 0 & 10 \end{matrix} = B$

c) $A - B = \begin{matrix} & M & CC & WB & S & KC \\ \# & 45 & 30 & 16 & 25 & 115 \end{matrix}$

3a) $\begin{matrix} & C & M & S & \text{cider} \\ \text{Oct} & 885 & 580 & 350 & 600 \\ \text{Dec} & 950 & 425 & 550 & 700 \end{matrix} = A$

b) $0.8A = \begin{matrix} & C & M & S & \text{cider} \\ O & 708 & 464 & 280 & 480 \\ D & 760 & 340 & 440 & 560 \end{matrix} = B$

#3

c) $\begin{matrix} & \$ \\ C & 0.25 \\ M & 0.50 \\ S & 0.30 \\ \text{cider} & 1.00 \end{matrix} = C$

d) $0.8A \times C$ or $B \times C = \begin{pmatrix} 708 & 464 & 280 & 480 \\ 760 & 340 & 440 & 560 \end{pmatrix} \begin{pmatrix} 0.25 \\ 0.50 \\ 0.30 \\ 1.00 \end{pmatrix}$

$= \begin{pmatrix} 708(0.25)+464(0.5)+280(0.3)+480(1) \\ 760(0.25)+340(0.5)+440(0.3)+560(1) \end{pmatrix} = \begin{pmatrix} 973 \\ 1052 \end{pmatrix}$

Total: $2025

#4 (A) uRent: $y = 20x + 20$

(B) DIY: $y = 23.5x + 8$

(C) RentAll: $y = 23x + 11$

A & B

$20x + 20 = 23.5x + 8$

$12 = 3.5x$

$x \approx 3.4$

B & C

$23.5x + 8 = 23x + 11$

$0.5x = 3$

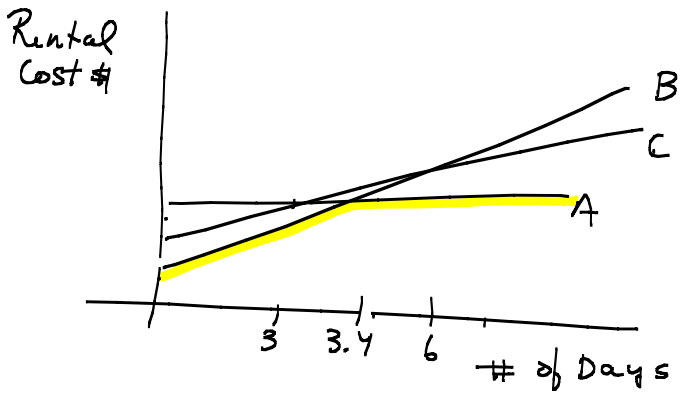
$x = 6$

A & C

$20x + 20 = 23x + 11$

$9 = 3x$

$3 = x$



If renting for less than 3.4 days use DIY. If renting for more than 3.4 days use uRent. Do not use RentAll.

Note Title 11/2/2009

4. (A) $y = 30x + 10$
(B) $y = 25x + 15$
(C) $y = 20x + 30$

y = total cost
x = # days.

A & B

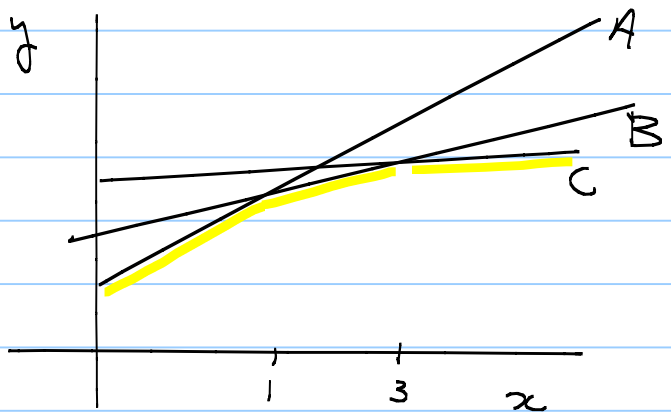
$30x + 10 = 25x + 15$
$5x = 5$
$x = 1$

B & C

$25x + 15 = 20x + 30$
$5x = 15$
$x = 3$

A & C

$30x + 10 = 20x + 30$
$10x = 20$
$x = 2$



If you can complete the job in one day (or less) use A. If you need the sander between 1 & 3 days (2 days) use B. If you need the sander more than 3 days, use C.

5a) $2x+y-z=-3$ ①
$5x+3y-2z=-5$ ②
$3x-y+z=-2$ ③

① $2x+y-z=-3$
② $3x-y+z=-2$
$5x=-5$
$x=-1$ ④

② $5x+3y-2z=-5$
(×2) ③ $6x-2y+2z=-4$
$11x+y=-9$ ⑤

⑤ $11x+y=-9$
$11(-1)+y=-9$
$-11+y=-9$
$y=2$

① $2x+y-z=-3$
$2(-1)+(2)-z=-3$
$-2+2-z=-3$
$-z=-3$
$z=3$

b) $x+2y=6 \Rightarrow x=6-2y$
$\frac{2}{5}x-4y=0$

$\therefore \frac{2}{5}(6-2y)-4y=0$ [mult by 5]

$2(6-2y)-4(5)y=0(5)$
$12-4y-20y=0$
$12-24y=0$
$12=24y$
$\frac{1}{2}=y$

$\therefore x=6-2(\frac{1}{2})$
$=6-1$
$x=5$

6) $75c+80u+1200L=41.50$ ①
$200c+100u+600L=62.00$ ②
$250c+150u+800L=83.00$ ③
$150c+50u+750L=44.50$ ④

You can use any three of these equations...

② $200c+100u+600L=62.00$
(×-2) ④ $-300c-100u-1500L=-89.00$
$-100c-900L=-27.00$ ⑤

③ $250c+150u+800L=83.00$
(×-3) ④ $-450c-150u-2250L=-133.50$
$-200c-1450L=-50.5$ ⑥

③ $250(0.18)+150u+800(0.01)=83.00$
$45+150u+8=83$
$150u+53=83$
$150u=30$
$u=0.20$

Local Calling: 1¢/min
Within Canada: 18¢/min
To USA: 20¢/min

(×-2) ⑤ $200c+1800L=54.00$
⑥ $-200c-1450L=-50.50$
$350L=3.50$
$L=0.01$

⑤ $-100c-900(0.01)=-27.00$
$-100c-9=-27$
$-100c=-18$
$c=0.18$

OR This could be solved using matrices.

① $75c + 80u + 1200L = 41.50$
② $200c + 100u + 600L = 62.00$
③ $250c + 150u + 800L = 83.00$

$$\underset{A}{\begin{bmatrix} 75 & 80 & 1200 \\ 200 & 100 & 600 \\ 250 & 150 & 800 \end{bmatrix}} \underset{X}{\begin{bmatrix} c \\ u \\ L \end{bmatrix}} = \underset{B}{\begin{bmatrix} 41.50 \\ 62.00 \\ 83.00 \end{bmatrix}}$$

Long Distance Canada → \$0.18/min
Long Distance USA → \$0.20/min
Local Calls → \$0.01/min

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[A]^-1[B]
          [[.18]
           [.2 ]
           [.01]]
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7a) h, \$
(2, 100)
(5, 130)

$m = \frac{\Delta y}{\Delta x} = \frac{130-100}{5-2} = \frac{30}{3} = 10$

$\therefore y = 10x + B$

(2, 100): $100 = 10(2) + B$
$100 = 20 + B$
$80 = B$

Equation: $y = 10x + 80$

b) Slope is \$10 per meter. This is the additional cost per meter of increasing the height of the sign. y-int is \$80. This is the cost of installing the sign at street level [no height]

c) $x = 8$: $y = 10(8) + 80$
$y = \$160$

∴ A sign installed 8m above the ground would cost \$160.

d) $y = 154$: $154 = 10x + 80$
$74 = 10x$
$7.4 = x$

∴ The sign is 7.4 m high if it cost \$154 to install

8a) Demand: (70, 1000)
(80, 900)

$m = \frac{900-1000}{80-70} = \frac{-100}{10} = -10$

(70, 1000) $y = -10x + B$
$1000 = -10(70) + B$
$1700 = B$

Demand Equation: $y = -10x + 1700$

b) Supply: (70, 300)
(80, 450)

$m = \frac{450-300}{80-70} = \frac{150}{10} = 15$

(70, 300): $y = 15x + B$
$300 = 15(70) + B$
$300 = 1050 + B$
$-750 = B$

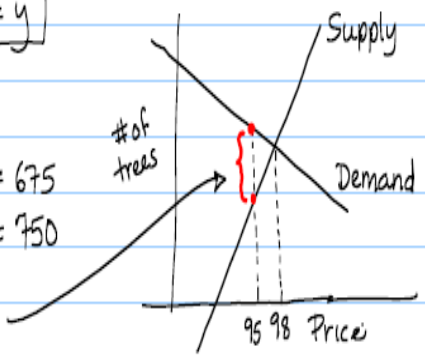
Supply Equation: $y = 15x - 750$

b) $-10x + 1700 = 15x - 750$
$2450 = 25x$
$98 = x$

$y = -10(98) + 1700 = 720 = y$

∴ Supply and demand are in equilibrium when price is \$98.

c) Supply: $y = 15(95) - 750 = 675$
Demand: $y = -10(95) + 1700 = 750$
∴ Shortfall of 75



d) Supply = 0 $\quad 0 = 15x - 750$

$750 = 15x$

$\boxed{x = 50}$

The supply would be eliminated when the price is \$50.00

e) $x = 0$ $\quad$ Demand: $y = -10(0) + 1700$

$y = 1700$

The consumer demand would be for 1700 trees.

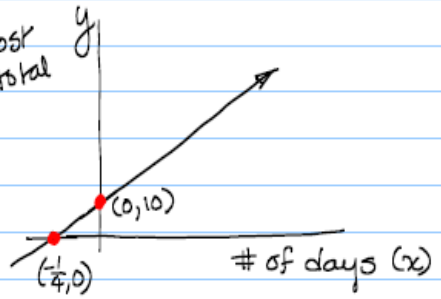
9a) $y = 40x + 10$

y-int: $(x = 0)$ $\boxed{y = 10}$

x-int: $(y = 0)$ $\quad 0 = 40x + 10$

$-10 = 40x$

$\boxed{-1/4 = x}$



The y-int is the fixed cost (or flat rate) of renting the wood splitter. That is, the cost of the insurances.

The x-int means if the cost was \$0, you could rent the splitter for <u>minus</u> $\frac{1}{4}$ days. Of course, this makes no sense, so the x-int has no meaning in this situation.

10. $3z + 2y + x = 6$

<u>x-int</u>

$3(0) + 2(0) + x = 6$

$x = 6$

$(6, 0, 0)$

<u>y-int</u>

$3(0) + 2y + (0) = 6$

$2y = 6$

$y = 3$

$(0, 3, 0)$

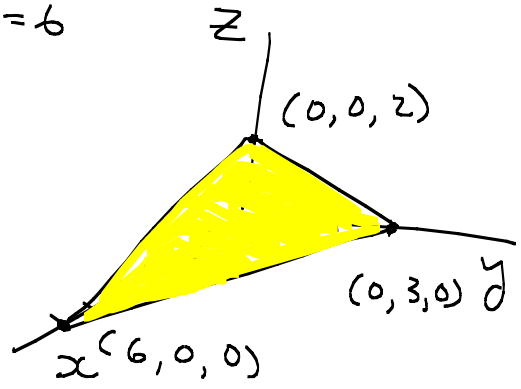
<u>z-int</u>

$3z + 2(0) + 0 = 6$

$3z = 6$

$z = 2$

$(0, 0, 2)$



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10a) $C = 0.2L + 0.05R + 10$

$L \rightarrow$ # of long distance min.
$R \rightarrow$ # of local prime time min
$C \rightarrow$ cost of plan

b) L-intercept $(C=0, R=0)$

$0 = 0.2L + 0.05(0) + 10$

$0 = 0.2L + 10$

$-0.2L = 10$

$L = -50$

R-intercept $(C=0, L=0)$

$0 = 0.2(0) + 0.05R + 10$

$0 = 0.05R = 10$

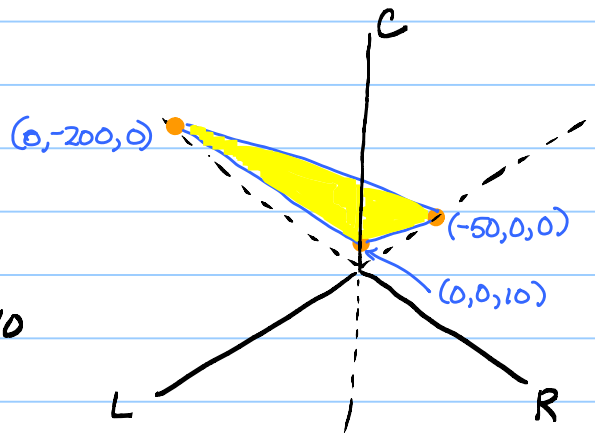
$-0.05R = 10$

$R = -200$

C-intercept $(L=0, R=0)$

$C = 0.2(0) + 0.05(0) + 10$

$C = 10$



11) $\begin{pmatrix} 2 & 1 & 4 \\ 3 & -1 & 6 \\ 5 & 3 & -2 \end{pmatrix} \cdot \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 9 \\ 11 \\ 17 \end{pmatrix}$

12) $y = ax^2 + bx + c$

$(10,130): 130 = 100a + 10b + c$

$(16,175): 175 = 256a + 16b + c$

$(22,58): 58 = 484a + 22b + c$

$\begin{pmatrix} 100 & 10 & 1 \\ 256 & 16 & 1 \\ 484 & 22 & 1 \end{pmatrix} \cdot \begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} 130 \\ 175 \\ 58 \end{pmatrix}$

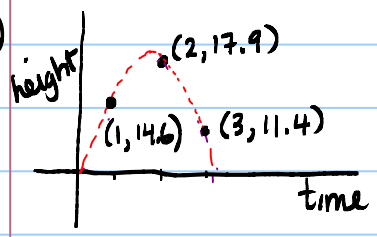
$\therefore A^{-1}C = \begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} -2.25 \\ 66 \\ -305 \end{pmatrix}$

$\therefore y = -2.25x^2 + 66x - 305$

If she charges \$20 she will be able to sell 115 t-shirts.

$$\begin{cases} y = -2.25(20)^2 + 66(20) - 305 \\ y = 115 \end{cases}$$

13)



$$y = ax^2 + bx + c$$

$$14.6 = a + b + c$$

$$17.9 = 4a + 2b + c$$

$$11.4 = 9a + 3b + c$$

$$\begin{pmatrix} 1 & 1 & 1 \\ 4 & 2 & 1 \\ 9 & 3 & 1 \end{pmatrix} \begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} 14.6 \\ 17.9 \\ 11.4 \end{pmatrix}$$

$$A^{-1}C = \begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} -4.9 \\ 18 \\ 1.5 \end{pmatrix}$$

$$\therefore y = -4.9x^2 + 18x + 1.5$$

After 3.5 seconds the stunt man is 4.475 m above the ground.

$$\begin{cases} y = -4.9(3.5)^2 + 18(3.5) + 1.5 \\ \quad = 4.475 \end{cases}$$